Physics 200B **Mechanics II** **Winter 2015**

Problem Set I: Due Thursday, January 15, 2015

1.) Determine the stable equilibrium positions for a simple pendulum which oscillates:

a.) horizontally, with $x = x_0 \cos\ \omega t$

b.) in a circle, with $x = r_0 \cos\ \omega t$, $y = r_0 \sin\ \omega t$.

Take $\omega >> \sqrt{g/\ell}$ and consider the full range of parameters.

2.) Now again consider a simple pendulum with support oscillating at $y = y_0 \cos\ \omega t$. If the pendulum has length ℓ (so $\omega_0 = \sqrt{g/\ell}$) and $\omega = 2\omega_0 + \in$, determine the conditions for, and growth rate of, parametric instability.

3.) Compute the threshold for parametric instability in the presence of linear frictional damping, as well as mismatch. For what range of mismatch $\in$ will instability occur?

4.) Let $H(q,p,t) = H_0(q,p) + V(q) d^2A/dt^2$ where $A(t)$ is periodic, with period $\tau << T$. Here T is the period of the motion governed by H_0 .

a.) Derive the mean field (i.e. short time averaged) equations for this system.

b.) Show that these mean field equations may be obtained from the effective Hamiltonian

$$K(p,q) = H_0(p,q) + \frac{1}{4m}\left\langle \left(\frac{dA}{dt}\right)^2 \right\rangle \left(\frac{\partial V(q)}{\partial q}\right)^2 .$$

Here < > means a short time average. You may assume $H_0 = p^2/2m + V_0(q)$.

5.) Consider the asymmetric top, with moments of inertia $I_1 < I_2 < I_3$. Here 1, 2, 3 refer to the principal axes in a frame for which the inertia tensor is diagonal. Using the Euler equations:

a.) Derive the equations of motion for $\Omega_1(t)$, $\Omega_2(t)$, $\Omega_3(t)$, the angular frequencies associated with axes 1, 2, 3.

b.) Show that if $\Omega_2 \cong \Omega_0$ while Ω_1, Ω_2 start from an infinitesimal perturbation, instability results. Show that $\Omega_1 \cong \Omega_0$ or $\Omega_3 \cong \Omega_0$ is stable.

c.) What are the two conserved quantities which constrain the evolution in b.)?

6.) Consider a free nonlinear oscillator which satisfied the equation

$$\ddot{x} + \omega_0^2 x = -\alpha x^2 - \beta x^3 .$$

Use Poincare-Linstedt perturbation theory to calculate the non-linear frequency shift and lowest order non-trivial solution.